

Some Aspects of Minimally Relativistic Newtonian Gravity*

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This paper aims to examine if the classical tests of General Relativity (GR) can be predicted by a simpler approach based on minimal changes in the Newtonian gravity. The approach yields a precession of the perihelion of Mercury by an amount $39.4''/\text{century}$ which is very close to the observed Dicke-Goldenberg value ($39.6''/\text{century}$), but less than the popularly accepted value ($43''/\text{century}$). The other tests exactly coincide with those of GR. Our analysis also displays the genesis as well as the role of geometry in the description of gravitational processes. The time dependent spherically symmetric equations, which are mathematically interesting, call for a further study. The model also allows unambiguous formulation of conservation laws. On the whole, the paper illustrates the limited extent to which a second rank tensor analogy (nonlinear) with flat background Faraday-Maxwell electrodynamics can be pushed in describing gravitation.

I. Introduction

A simple covariant model of gravitation based on minimal changes in the Newtonian Gravity is proposed here. It is shown to be fairly compatible with all the experimental tests of General Relativity (GR) besides having other interesting features. The approach is inspired mainly by the desire to examine how a nonlinear theory of gravitation on a flat background spacetime would look like, what are its possible results and to what extent does geometry play a role in the process of gravitational interaction. The analyses reported here are by no means complete. It is to be emphasized, once and for all, that the contents of this paper are not meant as an alternative to GR but only a heuristic exercise aimed at gaining some insight into the possible underlying physics of gravitation.

The proposed model can be categorized as a second rank tensor generalization of the idea, originally suggested by Bridgman [1] and subsequently developed as a vector model by Brillouin, Lucas [2], and Carstoiu [3], that there might be some deep similarity between electromagnetism and gravitation, that the field equations might be analogous to the electromagnetic equations but applicable to inertial matter. We shall have occasions to refer to Brillouin's works quite often.

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The contents of the paper are organized sectionwise as follows. In Sect. II, the field equations are described. Section III deals with equations of motion and the weak field limit. In Sect. IV, we solve the Schwarzschild problem and describe the classical tests of GR while Sect. V provides a brief sketch of some interesting features of the field equations including Birkoff's theorem. In Sect. VI, the conservation laws are formulated.

II. The Field Equations

The symmetric tensor potential $h_{\mu\nu}$ characterizing gravitation on a flat background satisfies the covariant field equations [4, 5]

$$\eta^{\alpha\beta} D_\alpha D_\beta h_{\mu\nu} = 16\pi \varrho_{\mu\nu} + \varkappa \eta^{\gamma\delta} D_\gamma h_{\mu\tau} D_\delta h^\tau_\nu, \quad (1)$$

where $\varrho_{\mu\nu}$ is the energy momentum tensor of matter from Special Relativity, $\eta^{\alpha\beta}$ represents a metric of the background flat spacetime, D_α is the covariant derivative with respect to η ; \varkappa is an adjustable dimensionless parameter, the tensor $h_{\alpha\beta}$ is raised or lowered with η . All Greek indices run from 0 to 3 while the Roman indices run from 1 to 3, $G = c = 1$.

The basic idea behind the construction of field equations (1) is the following. The Newtonian gravitational field has an energy density. But from Special Relativity it is known that such an energy density must be equivalent to a mass density that will produce its own field. Therefore, the field equations must be nonlinear. In the present model, the last term in (1), which is analogous to the covariant form of the elec-

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trostatic energy $(\nabla\phi)^2$, summarizes the nonlinearities of the gravitational field in terms of a contribution to the matter energy momentum tensor $\varrho_{\mu\nu}$, and acts as a secondary source. Except for this nonlinear term, the model mimics Faraday-Maxwell field (A^μ) theory. The parameter \varkappa measures the fraction by which the secondary source contributes to the field, and in that sense it can also be regarded as a kind of self coupling parameter.

It is to be noted in this connection that there exist in the literature many flat space theories of gravitation, notably the most recent one by Logunov et al. [6], which has been proposed as an alternative to GR. There are also arguments showing that all flat space spin-2 long range field theories of gravitation eventually lead to Einstein's GR equations [7, 8]. As far as the present model is concerned, it does not go over into GR equations because of the presence of the $(\text{grad})^2$ term on the r.h.s. of the field equations (1). In this sense, it is a truly non-GR model.

It must be mentioned that the field equations (1) were originally proposed by Biswas [4]. However, subsequently an error in the algebraic computation of Biswas was pointed out by Peters [5]. In this paper, we retain only the field equations (1), change the coupling and discuss the consequences in fair detail.

III. The Equations of Motion and Weak Field Limit

The mass shell equation for a particle of rest mass m_0 in the absence of gravitation is given by

$$\eta^{\mu\nu} p_\mu p_\nu + m_0^2 = 0, \quad (2)$$

where p^μ is the momentum 4-vector. This equation would obviously change when a tensor potential is introduced. The introduction of a vector potential A^μ as a minimal coupling term is wellknown from electrodynamics. It is given by the transition $p^\mu \rightarrow p^\mu + A^\mu$. The introduction of a tensor potential $h_{\alpha\beta}$ likewise induces a minimal coupling through the transition

$$\eta_{\mu\nu} \rightarrow \eta_{\mu\nu} - (h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h) = a_{\mu\nu}, \quad (3)$$

where $h_{\mu\nu}$ need not be small. The choice of minimal coupling through the terms linear in h is a part of the general idea of making minimal changes in the Newtonian gravity while introducing general covariance. Therefore, in a tensor potential, the mass shell equation should generalize to

$$a^{\mu\nu} p_\mu p_\nu + m_0^2 = 0. \quad (4)$$

For convenience, we use the Hamilton-Jacobi equation for studying the motion of test objects with rest mass m_0 :

$$a^{\mu\nu} \frac{\partial S}{\partial x^\mu} \frac{\partial S}{\partial x^\nu} + m_0^2 = 0, \quad (5)$$

where $p_\mu = \frac{\partial S}{\partial x^\mu}$. For photons, we use the eikonal equation

$$a^{\mu\nu} \frac{\partial \psi}{\partial x^\mu} \frac{\partial \psi}{\partial x^\nu} = 0. \quad (6)$$

Before proceeding further, some remarks concerning the nature of the tensor $a^{\mu\nu}$ are in order. Is it just an ordinary tensor defined in a space having the flat metric $\eta^{\mu\nu}$ or is it itself a metric tensor defining a Riemannian spacetime? It is obvious that this question can not be answered from within our theoretical framework alone since the tensor $a^{\mu\nu}$ is the result of a coupling between two tensors $\eta^{\mu\nu}$ and $h^{\mu\nu}$ having different roles. Therefore, the logical step is to look towards the empirical facts. But which empirical facts? As we are proposing herein a model for gravitation, it is only reasonable that we reset our clock back to 1915 and consider only those facts that were available before that date but remained unexplained. A singular candidate for this is the fact of anomalous perihelic precession of Mercury, which was known at least since 1765. In fact, this fact always stood as a check against which all variants of gravitation theories were weighed. Einstein's GR not only passed this test but also predicted other phenomena which were subsequently confirmed experimentally. In order to pass a similar test, let us exercise the two options with $a^{\mu\nu}$ implied in the question. First, if we treat $a^{\mu\nu}$ as an ordinary tensor, that is, raise or lower it with η , and integrate the equation of motion (5), we obtain a value for perihelic precession which is much smaller than the empirical value (43"/century for Mercury). Hence this option is ruled out on empirical grounds. The second option with the $a^{\mu\nu}$ is to interpret it as a "metric tensor" of an "effective" Riemannian space so that $a^{\mu\nu} a_{\mu\tau} = \delta_\tau^\mu$. We shall then find that (5) gives a value for precession 39.4"/century, which is much nearer to the empirical value. Therefore a metric tensor interpretation is somewhat forced on $a^{\mu\nu}$ in order that (5) yields results which are empirically verified. One can here equally perceive the genesis of a Riemannian metric from the coupling equation (3). Way back in 1970, adopting a different approach, Deser [9] rigorously

confirmed this basic result, that is, the natural emergence of a Riemannian geometry as a result of universal coupling. It becomes also clear that a test particle can only recognize a Riemannian space with metric $a^{\mu\nu}$ and the initial flat space metric $\eta^{\mu\nu}$ becomes “unobservable”. This last result is in perfect harmony with the philosophy expressed by Deser [9], and rather lately by Zel’dovich and Grishchuk [8].

The genesis and the role of the tensor $a^{\mu\nu}$ has thus become more transparent, and it follows as a corollary that an a priori postulation of a Riemannian geometry satisfying some arbitrary field equations is not at least essential. Let us now return to the equations of motion (4).

In the weak field limit, (4) yields [4]:

$$\dot{p}_j \cong -\frac{m_0}{4} \frac{\partial h_{00}}{\partial x^j}.$$

Comparing the above with $\dot{p}_j = -m_0 \frac{\partial \Phi}{\partial x^j}$, where Φ is the Newtonian potential, we find that

$$h_{00} \cong 4\Phi = -\frac{4M_0}{r}, \quad (7)$$

where $M_0 = 4\pi \int_0^a \varrho(r) r^2 dr$ is the mass of the gravitating object with radius a . The same weak field limit also obtains if we make appropriate approximations right in the field equations. With all the above at hand, let us now proceed to find the solutions of the field equations (1).

IV. Solution of the Schwarzschild Problem

We seek a spherically symmetric static solution of the matter free equations

$$\eta^{\alpha\beta} D_\alpha D_\beta h_{\mu\nu} = \varkappa \eta^{\gamma\delta} D_\gamma h_{\mu\tau} D_\delta h_\nu^\tau. \quad (8)$$

Let us adopt the standard spherical polar coordinates (t, r, θ, φ) and the metric $\eta_{\alpha\beta} = (-1, 1, r^2, r^2 \sin^2 \theta)$. The static and spherically symmetric nature of the problem imply that all components of $h_{\mu\nu}$ except h_{00} and h_{11} vanish. Furthermore, the structure of (8) implies that $h_{11} = 0$ [5]. Therefore, the only nontrivial equation accruing from (8) is

$$\frac{1}{r^2} \frac{d}{dr} \left[r^2 \frac{dh_{00}}{dr} \right] = -\varkappa \left(\frac{dh_{00}}{dr} \right)^2.$$

Notice that this equation is similar to Brillouin’s equation for gravistatics [2]. Rewriting the above,

$$\frac{1}{r^2} \frac{d}{dr} \left[r^2 \frac{d e^{\varkappa h_{00}}}{dr} \right] = 0. \quad (9)$$

Utilizing the weak field condition (7) and that $h_{00} \rightarrow 0$ as $r \rightarrow \infty$, we find that

$$h_{00} = \frac{1}{\varkappa} \ln \left(1 - \frac{4\varkappa M_0}{r} \right). \quad (10)$$

Using (3), we obtain

$$a_{00} = - \left[1 + \frac{1}{2\varkappa} \ln \left(1 - \frac{4\varkappa M_0}{r} \right) \right], \quad (11)$$

$$a_{ii} = \eta_{ii} \left[1 - \frac{1}{2\varkappa} \ln \left(1 - \frac{4\varkappa M_0}{r} \right) \right]. \quad (12)$$

For our purposes, we expand a_{00} and a_{ii} , retaining terms only up to $\frac{1}{r^2}$:

$$(a^{00})^{-1} = a_{00} \cong - \left(1 - \frac{2M_0}{r} - \frac{4\varkappa M_0^2}{r^2} \right), \quad (13)$$

$$(a^{ii})^{-1} = a_{ii} \cong \eta_{ii} \left(1 + \frac{2M_0}{r} + \frac{4\varkappa M_0^2}{r^2} \right). \quad (14)$$

Notice that the second order terms have not been imported via nonlinear coupling with extra unknown, physically meaningless parameters. Those terms are automatically contributed by the logarithmic function with a single parameter \varkappa , which here has a definite physical meaning. To some extent, the second order term resembles the contribution due to charge in the Reissner-Nordström solution of GR. We shall naturally be interested to study the effect of this term in the sequel.

Turning now to the various tests of gravitation, it is straightforward to compute through (4), (6), (13), and (14) that the wellknown formulae for the gravitational red shift and bending of light rays follow. Since none of these tests involves \varkappa significantly, we see from (13) and (14) that we are effectively dealing with what may be called the Schwarzschild solution in isotropic form (for large distances). Consequently, the results are as expected and we do not wish to reproduce them here. Instead, we are interested to see the effect of \varkappa on the precession of the perihelion of Mercury.

Consider (5) and customarily define [10], for $\theta = \frac{\pi}{2}$,

$$S = -E_0 t + L \varphi + S_r(r), \quad (15)$$

where E_0 and L are the constant energy and angular momentum, respectively, of the test object, here Mercury. Then from (5) it follows that

$$S_r = \int \left[\left(1 - \frac{2M_0}{r} - \frac{4\kappa M_0^2}{r^2} \right)^{-1} \cdot \left(1 - \frac{2M_0}{r} - \frac{4\kappa M_0^2}{r^2} \right) E_0^2 - m_0^2 \left(1 + \frac{2M_0}{r} + \frac{4\kappa M_0^2}{r^2} \right) - \frac{L^2}{r^2} \right]^{1/2} dr.$$

With Landau and Lifshitz [10], substitute $r = r' + M_0$ so that

$$1 - \frac{2M_0}{r} \cong 1 - \frac{2M_0}{r'} + \frac{2M_0^2}{r'^2} \quad (16)$$

and

$$1 + \frac{2M_0}{r} \cong 1 + \frac{2M_0}{r'} + \frac{2M_0^2}{r'^2}. \quad (17)$$

Using $E_0^2 = (m_0 + E')^2$, where E' is the nonrelativistic part of the energy, and retaining terms up to $\frac{1}{r'^2}$, we have, dropping primes on r :

$$S_r = \int \left[(2E'm_0 + E'^2) + \frac{1}{r} (2m_0^2 M_0 + 8M_0 m_0 E') - \frac{1}{r^2} \{ L^2 - 6m_0^2 M_0^2 (1 + \frac{2}{3}\kappa) \} \right]^{1/2} dr. \quad (18)$$

By actual numerical computation it follows that the term $6m_0^2 M_0^2 (1 + \frac{2}{3}\kappa)$ corresponds to a perihelic precession of exactly 43"/century if $\kappa = 0$. In this case,

$$\lim_{\kappa \rightarrow 0} h_{00} = -\frac{4M_0}{r}. \quad (19)$$

Consequently, neither h_{00} nor $a_{\mu\nu}$ exhibit any singularity anywhere except at $r = 0$. Also, the contribution of the nonlinear secondary source to the gravitational field is then altogether disregarded. From the physical point of view, this neglect does not appear satisfactory if we accept, as we do, that any form of energy creates a gravitational field of its own. Therefore, we must keep open the possibility that $\kappa \neq 0$, in which case, depending on the numerical value of κ , there will result a small deviation from the value 43". We have to wait till Sect. VI where we find a value of κ using a novel result of Brillouin [2].

The foregoing results could be obtained because of the specific form of the set of equations (1). However, it must be stated that no claim is made here of the uniqueness of the form of field equations (1). These

field equations have been primarily dictated by simple physical considerations. There is always a scope for improving upon the set of field equations (1) based on either physical or mathematical considerations. For example, we can introduce a polynomial with arbitrary coefficients in place of $e^{\kappa h_{00}}$ in (9) and then examine how the parent equations should thereby modify. We could as well consider in this paper only a linear theory ($\kappa = 0$) which would still be consistent with experimental results, but then the nonlinearity of the field equations will be lost.

Having described the classical tests of GR within the present framework, we now proceed to discuss the time dependent solution.

V. Time Dependent Solution

We consider a slightly more general case by introducing time dependence into the above Schwarzschild problem. The potential h_{00} now satisfies the equation (other components of $h_{\mu\nu}$ are zero)

$$\begin{aligned} \frac{1}{r^2} \frac{\partial}{\partial r} \left[r^2 \frac{\partial h_{00}}{\partial r} \right] - \frac{\partial^2 h_{00}}{\partial t^2} \\ = -\kappa \left[\left(\frac{\partial h_{00}}{\partial r} \right)^2 - \left(\frac{\partial h_{00}}{\partial t} \right)^2 \right]. \end{aligned} \quad (20)$$

The equation can be rewritten as

$$\square^2 e^{\kappa h_{00}(r,t)} = 0, \quad \kappa \neq 0, \quad (21)$$

where

$$\square^2 \equiv \frac{1}{r^2} \frac{\partial}{\partial r} \left[r^2 \frac{\partial}{\partial r} \right] - \frac{\partial^2}{\partial t^2}. \quad (22)$$

The solution of (21) under usual conditions is

$$h_{00} = \frac{1}{\kappa} \ln \left[1 + \frac{1}{r} \{ F(r - ct) + G(r + ct) \} \right], \quad \kappa \neq 0. \quad (23)$$

where F and G are arbitrary functions.

The irremovable time dependence in h_{00} indicates that Birkoff's theorem of GR does not hold here. This is not unexpected because the present approach is fashioned after Faraday-Maxwell theory, where radial oscillations of charged spheres give rise to E -waves.

In the far field (r large),

$$h_{00} \cong \frac{1}{\kappa r} [F(r - ct) + G(r + ct)], \quad \kappa \neq 0. \quad (24)$$

representing spherical wavefronts. If $\kappa = 0$, we get the usual wave equation in h_{00} from (20) throughout the

3-space. However, we must view the above results as more in the nature of a limitation of the model in question than anything else. Below we display another mathematically interesting aspect of the field equations (1).

Let us include also the matter part $\varrho_{\mu\nu}$ in the h_{00} equation under spherical symmetry. Choosing the centre-of-mass coordinate system such that the 4-velocities are

$$U_0 = 1, \quad U_i = 0,$$

we get, from field equations (1),

$$\begin{aligned} \square^2 h_{00} + \kappa \left[\left(\frac{\partial h_{00}}{\partial r} \right)^2 - \left(\frac{\partial h_{00}}{\partial t} \right)^2 \right] \\ = 16\pi \varrho_{00} = 16\pi \varrho(r, t). \end{aligned} \quad (25)$$

We can rewrite it as

$$(\square^2 - 16\pi \varrho \kappa) H(r, t) = 0, \quad \kappa \neq 0, \quad (26)$$

where $H(r, t) = e^{\kappa h_{00}(r, t)}$. The quantity $h_{00}(r, t)$ represents a gravitational field component inside a matter distribution characterized by a density distribution $\varrho(r, t)$. The equation (26) resembles Klein-Gordon equation for the function $H(r, t)$, and it can be solved by substituting

$$H(r, t) = v(r) g(t) \quad (27)$$

such that

$$\ddot{g} + \lambda g = 0 \quad (28)$$

where \cdot represents time derivative and

$$r^2 v'' + 2r v' - q(r) v = 0, \quad (29)$$

where $'$ represents space derivative and

$$q(r) = r^2 [16\pi \kappa \varrho(r) - \lambda],$$

λ is the eigenvalue. It is easy to recognize that (28) and (29) are the usual Sturm-Liouville eigenvalue problem [11] resulting in a periodic solution for $H(r, t)$.

The equation for the gravitational potential component $h_{11}(r, t)$ can be obtained from (1) as

$$\begin{aligned} \square^2 h_{11} - \frac{4h_{11}}{r^2} - \kappa \left[\left(\frac{\partial h_{11}}{\partial r} \right)^2 - \left(\frac{\partial h_{11}}{\partial t} \right)^2 + \frac{2h_{11}}{r^2} \right] \\ = 16\pi p(r, t) \end{aligned} \quad (30)$$

while h_{22} and h_{33} equations yield only one equation:

$$2h_{11} - \kappa h_{11}^2 = 16\pi r^2 p(r, t), \quad (31)$$

where $p = p(r, t)$ is the pressure distribution inside matter. p and q are connected by an equation of state,

as usual. It is interesting to see that $p(r, t)$ can be altogether eliminated from (30) using (31). An exact solution of (30) and its implications will be discussed elsewhere. However, such interior solutions are only of academic interest as also in GR.

It can also be shown by customary analysis that the gauge invariance and the existence of maximal Killing symmetries leave only two components of $h_{\alpha\beta}$ as independent, corresponding to helicities ± 2 , in the general case of the plane wave approximation of (1), which are

$$\eta^{\alpha\beta} D_\alpha D_\beta h^{\mu\nu} = 16\pi \varrho^{\mu\nu}. \quad (32)$$

This result is of course compatible with GR. We shall now address ourselves to the formulation of conservation laws.

VI. Conservation Laws

Consider the field equations (1) and add on both sides a tensor $X^{\mu\nu}$ involving $h^{\mu\nu}$:

$$X^{\mu\nu} = D_\alpha D_\beta [\eta^{\mu\nu} h^{\alpha\beta} - \eta^{\beta\mu} h^{\alpha\nu} - \eta^{\beta\nu} h^{\alpha\mu}], \quad (33)$$

so that the field equations remain undisturbed. Then the l.h.s. of (1) becomes

$$J^{\mu\nu} = D_\alpha D_\beta [\eta^{\beta\nu} h^{\alpha\mu} + \eta^{\mu\beta} h^{\alpha\nu} - \eta^{\mu\nu} h^{\alpha\beta} - \eta^{\alpha\beta} h^{\mu\nu}], \quad (34)$$

which satisfies the identity

$$D_\nu J^{\mu\nu} = 0. \quad (35)$$

Consequently, we obtain, from the r.h.s. of (1)

$$D_\nu \tau^{\mu\nu} = 0 \quad (36)$$

where

$$\tau^{\mu\nu} = 16\pi \varrho^{\mu\nu} + \kappa \eta^{\gamma\delta} D_\gamma h^{\mu\tau} D_\delta h_\tau^\nu + X^{\mu\nu}. \quad (37)$$

The equations (36) represent the four differential conservation laws. The symmetric tensor $\tau^{\mu\nu}$ represents the energy momentum tensor of a system composed of matter and gravitational field. We are not considering any contribution from the so-called vacuum energy for the moment.

To obtain the integral conservation laws, observe that our background spacetime, being flat, admits a (maximal) ten parameter group of motions. By considering the Killing equations, it is straightforward to show that, for the translation subgroup, the total energy momentum 4-vector is conserved:

$$P^\mu = \frac{1}{16\pi} \int \tau^{0\mu} \sqrt{-\eta} d^3x = \text{constant}. \quad (38)$$

Also, corresponding to the rotation subgroup, the total angular momenta are conserved:

$$M^{\mu\nu} = \frac{1}{16\pi} \int (x^\nu \tau^{0\mu} - x^\mu \tau^{0\nu}) \sqrt{-\eta} d^3x = \text{constant}. \quad (39)$$

The quantity P^0 represents the total energy (=mass) of a system composed of matter and gravitation. Let us compute this for the Schwarzschild problem of Section IV. From (10) it follows that

$$\frac{dh_{00}}{dr} = \frac{4M_0}{r^2} \left(1 - \frac{4\kappa M_0}{r}\right)^{-1}$$

or, expanding

$$\left(\frac{dh_{00}}{dr}\right)^2 = \frac{16M_0^2}{r^4} \left[1 + \frac{8\kappa M_0}{r} + \dots\right]. \quad (40)$$

Then, for a gravitating object of radius $r = a$, we can straightforwardly compute from (37) and (38) that, to first order,

$$P^0 = M_0 + \frac{4\kappa M_0^2}{a}, \quad (41)$$

where

$$M_0 = 4\pi \int_0^a \varrho(r) r^2 dr.$$

One immediately notices that, in the limit, as $a \rightarrow \infty$,

$$P^0 = M_0. \quad (42)$$

The quantity P^0 is the “inertial mass” and M_0 is the “gravitational mass” of the system in question. The equation

$$P^0 = M_0 \left(1 + \frac{4\kappa M_0}{a}\right)$$

has a remarkable similarity with the expression for the total inertial mass obtained via the full use of GR equations [12]

$$P_0 = M_0 \left(1 - \frac{M_0}{2a}\right). \quad (43)$$

Obviously, we can not use this to find our value of κ since we are not permitted to presuppose GR. However, there is a way out. Proceeding from purely classical considerations, Brillouin showed that the total inertial mass M_t of a composite system (the mass M_0 plus the contribution to mass from the generated

gravistatic field) is, to first order,

$$M_t = M_0 \left[1 - \frac{M_0}{2a}\right]. \quad (44)$$

If we equate the two inertial masses P^0 and M_t , we immediately find from (41) and (44) that $\kappa = -1/8$.

It can further be verified that

$$P^i = 0, \quad M^{\mu\nu} = 0, \quad (45)$$

as expected. The results embodied in (42) and (45) remain unchanged whatever background coordinate mesh we choose.

With $\kappa = -\frac{1}{8}$, the perihelic shift of Mercury $\delta\phi$ gets modified by a factor $\frac{11}{12}$, that is, to a numerical value of 39.4"/century while the results of other tests do not modify appreciably. It is remarkable that this value of $\delta\phi$ is so very close to the value observed by Dicke and Goldenberg [13]. It must, however, be remembered that some controversy surrounds the Dicke-Goldenberg value. Hence, it appears safer to consider the value $\delta\phi \sim 43''/\text{century}$ observed by others as the correct value and regard the present model as inferior to GR.

Note further from (10), (11), and (12), that both $h_{\mu\nu}$ and $a_{\mu\nu}$ display a logarithmic singularity at

$$r = r_g = \frac{M_0}{2}. \quad (46)$$

This is precisely the singular radius in GR for the Schwarzschild metric in isotropic coordinates. If κ were zero, we would not obtain this singular radius. This shows that the presence of κ in the field equations is required at least for ensuring a singularity!

As an immediate step towards further progress, it would be worthwhile to study the largely academic Kerr problem within the framework of the proposed model. It would also be worthwhile to study some earlier results [14–16] in the light of the present approach. The latter, of course, has its own evident limitations. Nevertheless, it is hoped that it provides a clearer physical insight and illustrates the extent to which a second rank tensor analogy can be drawn with electrodynamics in describing gravitation.

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